data in Table I, reference 2, at the trial value of r_0 permits one to obtain for each value of τ_0 a corresponding value of α_0. The calculation of F_0 then follows: (2) for each value of α in Table I, reference 2, the values of $(\partial\tau/\partial r)_\alpha$ were evaluated[28] at the trial value of r_0. Using the interpolation tables one gets from these figures (*i.e.*, α, $\partial\tau/\partial r$ values) a value of $(\partial\tau/\partial r)_\alpha$ for each value of α_0 obtained in (1) above: (3) at the trial value of r_0 one gets from the data in Table I, reference 2, the values[28] of $(\partial\tau/\partial\alpha)_r$ for each value of α. Using the interpolation tables one gets from these figures (*i.e.*, τ, $\partial\tau/\partial\alpha$ values) a value of $(\partial\tau/\partial\alpha)_{r_0}$ for each value of τ_0 obtained in (1) above; (4) calculations of $(F_r)_0$ and $(F_k)_0$ follow in accordance to their definitions.

The normal equations for computing the small correction terms and k' are written with detached coefficients as

$$\begin{array}{ccc} r' & k' & 1 \\ \Sigma(F_r)^2_0 & \Sigma(F_r)_0(F_k)_0 & \Sigma(F_r)_0F_0 \\ & \Sigma(F_k)^2_0 & \Sigma(F_k)_0F_0 \end{array} \qquad (22)$$

Through use of the tabular scheme in Deming (p. 158) one can calculate the values of r' and k'. Standard deviation from the measurements and the standard error of the parameters readily are obtained using the figures in the tabular scheme.

In treating the data of run 5, we selected $r_0 = 1.50$ and $k_0 = 5.331$ as our trial values. In calculating τ_0 at each experimental point the value of t used was defined as

$$t = t_{\text{exp}} - t_{\text{intercept}}$$

where $t_{\text{intercept}}$ was the time axis intercept of our straight line plot. For run 5, $t_{\text{intercept}}$ was 4.45 min. Some of the essential calculated results are given in Table I. r' and k' turned out to be 0.015 and 0.07, respectively, and the standard estimated errors in r and k were 0.08 and 0.13, respectively.

COLLEGE PARK, MARYLAND

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[28] For this purpose we used the "Tables of Coefficients for Obtaining the First Derivatives Without Differences," National Bureau of Standards, Applied Mathematics Series, 2, 1948.

[CONTRIBUTION FROM THE DEPARTMENT OF CHEMISTRY, FLORIDA STATE UNIVERSITY AND RESEARCH INCORPORATED]

On the Solvent Isotope Effect of Deuterium in Aqueous Acid Solutions

BY E. LEE PURLEE[1]

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The Gross–Butler theory for the solvent isotope effect of deuterium on acid-catalyzed reaction rates in D_2O–H_2O mixtures is re-examined. New values for the thermodynamic activity function, $Q'(n)$, for proton and deuteron at 25° are derived from experimental data by using the correct value for the isotopic exchange equilibrium constant for D_2O–H_2O and by taking into account the apparent difference in free energy between (Cl^-) in D_2O and (Cl^-) in H_2O. An independent check of the $Q'(n)$-function at 25° is obtained by applying the rule of the geometric mean to the distribution of hydrogen isotopes between lyonium ion and water. Values for the thermodynamic activity function, $Q'(n)$, are calculated for application at temperatures other than 25°, and it is demonstrated that equilibrium constants and reaction rates observed at 0, 15, 18, 30 and 35° are well predicted by the Gross–Butler theory when values of the $Q'(n)$-function corresponding to the appropriate temperatures are used. The nature of deviations from the Gross–Butler rate equation is briefly discussed. It is noted that the acid-catalyzed inversion of sucrose and the acid-catalyzed hydrolysis of methyl acetate and ethyl formate show significant deviations from the theory. The complete failure of the Gross–Butler rate equation to predict the rates for the acid-catalyzed mutarotation of glucose indicates that this reaction does not proceed *via* a pre-equilibrium mechanism.

The Gross–Butler Theory

The theory for the solvent isotope effect of deuterium on reaction rates and equilibria in aqueous acid solutions was originally proposed by Gross and co-workers[2–4] in 1936. Shortly thereafter, Butler and co-workers[5–7] further developed the theory.[8] It has been employed by various investigators as a diagnostic aid in the elucidation of reaction mechanisms. Recently, Purlee and Taft[9] applied the theory to the acid-catalyzed hydration of olefins and Pritchard and Long[10] applied it to the acid-catalyzed hydrolysis of ethylene oxide and some of its derivatives.

Butler and his associates[6,7] derived functions relating the activities of protons and deuterons in D_2O–H_2O mixtures to proton activities in H_2O from experimental and theoretical values of equilibrium constants between the various isotopic species in acidified D_2O–H_2O mixtures. Since the value of one of the theoretical equilibrium constants (*i.e.*, K of eq. 2) used by Nelson and Butler has been shown to be incorrect[7,11,12] and since the value of a second equilibrium constant (L of eq. 4) derived indirectly from experimental data is open to serious question,[13,14] it is of importance to re-examine the treatment of available data.

The Gross–Butler Rate Equation.—As previously discussed,[9] this equation applies to acid-catalyzed reactions involving a proton transfer preliminary to the rate-determining step. However, in order to avoid the assumption that lyonium ion activities

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[1] Department of Chemistry, Florida State University. Requests for reprints should be sent to the author at P. O. Box 206, Tallahassee, Florida.

[2] Ph. Gross, H. Steiner and F. Krauss, *Trans. Faraday Soc.*, **32**, 877 (1936).

[3] Ph. Gross and A. Wischen, *ibid.*, **32**, 879 (1936).

[4] Ph. Gross, H. Steiner and H. Suess, *ibid.*, **32**, 883 (1936).

[5] J. C. Hornel and J. A. V. Butler, *J. Chem. Soc.*, 1361 (1936).

[6] W. J. C. Orr and J. A. V. Butler, *ibid.*, 330 (1937).

[7] W. E. Nelson and J. A. V. Butler, *ibid.*, 958 (1938).

[8] The theory has been referred to as that of Nelson and Butler (*cf.* ref. 9 and 10). If names are to be associated with the theory, that of Gross should indeed be included.

[9] E. L. Purlee and R. W. Taft, Jr., THIS JOURNAL, **78**, 5807 (1956).

[10] J. G. Pritchard and F. A. Long, *ibid.*, **78**, 6008 (1956).

[11] B. Topley and H. Eyring, *J. Chem. Phys.*, **2**, 220 (1934).

[12] H. C. Urey, *J. Chem. Soc.*, 562 (1947).

[13] G. Schwarzenbach, *Z. Elektrochem.*, **44**, 302 (1938).

[14] G. Schwarzenbach, A. Epprecht and H. Erlenmeyer *Helv. Chim. Acta*, **19**, 1292 (1936).

are independent of isotopic solvent composition, eq. 5 of ref. 9 should be replaced by

$$\frac{k_n}{k_H} = \frac{(a_H{}^+)_n(f_*)_H}{(a_H{}^+)_H(f_*)_n} + \frac{k_D}{k_H}\frac{(a_D{}^+)_n(f_*)_H}{(a_H{}^+)_H(f_*)_n} \qquad (1)$$

$(a_H{}^+)$ = activity of the proton in H_2O
$(a_D{}^+)$ = activity of the deuteron in D_2O
n = (D)/(H + D) = atom fraction of deuterium in the solvent
k_H, k_n, k_D = obsd. reaction rate constants in H_2O, in D_2O–H_2O mixtures of deuterium fraction n, and in D_2O, respectively
f_* = activity coefficient of the reaction transition state

Pertinent equilibria between the various solvent species, protons and deuterons along with their equilibrium constants are

$$H_2O + D_2O = 2HDO,\ K \qquad (2)$$

$$\left.\begin{array}{l} H_2O + H^+ = H_3O^+,\ k_1{}^{+H} \\ H_2O + D^+ = H_2DO^+,\ k_1{}^{+D} \\ HDO + H^+ = H_2DO^+,\ k_2{}^{+H} \\ HDO + D^+ = HD_2O^+,\ k_2{}^{+D} \\ D_2O + H^+ = HD_2O^+,\ k_3{}^{+H} \\ D_2O + D^+ = D_3O^+,\ k_3{}^{+D} \end{array}\right\} \qquad (3)$$

$$2D_3O^+ + 3H_2O = 2H_3O^+ + 3D_2O,\ L \qquad (4)$$

The proton and deuteron activities are obtained by setting $k_1{}^{+H} = 1$, and $k_3{}^{+D} = 1$

$$\left.\begin{array}{l} (a_H{}^+) = a_{H_3O^+}/a_{H_2O} \\ (a_D{}^+) = a_{D_3O^+}/a_{D_2O} \end{array}\right\} \qquad (5)$$

In an acidified D_2O–H_2O mixture, the sum of the lyonium ion activities is given by

$$a_{M_3O^+} = a_{H_3O^+} + a_{H_2DO^+} + a_{HD_2O^+} + a_{D_3O^+} \qquad (6)$$

The individual lyonium ion activities may be expressed in terms of the appropriate equilibrium constants of equations 3; and equation 6 can be written

$$a_{M_3O^+} = a_{H^+}\left(a_{H_2O} + a_{HDO}k_2{}^{+H} + a_{D_2O}k_3{}^{+H} + \frac{a_{D^+}a_{D_2O}}{a_{H^+}}\right) \qquad (7)$$

From equations 5 and 4

$$\frac{a_{D^+}}{a_{H^+}} = \frac{(a_{D_2O})^{1/2}}{(a_{H_2O})^{1/2}} \times \frac{1}{(L)^{1/2}} \qquad (8)$$

hence

$$a_{M_3O^+} = a_{H^+}\left[a_{H_2O} + a_{HDO}k_2{}^{+H} + a_{D_2O}k_3{}^{+H} + \frac{(a_{D_2O})^{3/2}}{(a_{H_2O})^{1/2}} \times \frac{1}{(L)^{1/2}}\right] \qquad (9)$$

and

$$a_{M_3O^+} = \frac{a_H{}^+Q'(n)}{(a_{H_2O})^{1/2}} \qquad (10)$$

where the function $Q'(n)$ is defined by

$$Q'(n) = \left[(a_{H_2O})^{3/2} + (a_{H_2O})^{1/2}(a_{HDO})k_2{}^{+H} + (a_{H_2O})^{1/2}(a_{D_2O})k_3{}^{+H} + \frac{(a_{D_2O})^{3/2}}{(L)^{1/2}}\right] \qquad (11)$$

The proton and deuteron activities are given by

$$\left.\begin{array}{l} a_H{}^+ = \dfrac{a_{M_3O^+}(a_{H_2O})^{1/2}}{Q'(n)} \\ a_D{}^+ = \dfrac{a_{M_3O^+}(a_{D_2O})^{1/2}}{Q'(n)(L)^{1/2}} \end{array}\right\} \qquad (12)$$

On the assumption that the activity coefficients, f_+, of the various lyonium ion species are equivalent at a given acid concentration, the proton and deuteron activity ratios of equation 1 are given by

$$\left.\begin{array}{l} \dfrac{(a_{H^+})_n}{(a_{H^+})_H} = \dfrac{(a_{H_2O})_n{}^{1/2}(f_+)_n}{Q'(n)(f_+)_H} \\ \dfrac{(a_{D^+})_n}{(a_{H^+})_H} = \dfrac{(a_{D_2O})_n{}^{1/2}(f_+)_n}{Q'(n)(L)^{1/2}(f_+)_H} \end{array}\right\} \qquad (13)$$

Equation 1 can thus be reduced to

$$\frac{k_n}{k_H} = \frac{1}{Q'(n)}\left[(a_{H_2O})_n{}^{1/2} + \frac{k_D(a_{D_2O})_n{}^{1/2}}{k_H(L)^{1/2}}\right] \qquad (14)$$

provided the activity coefficient ratio, f_+/f_*, is assumed to be independent of isotopic solvent composition. This is a reasonable assumption since the reaction transition state is a univalent, positive species.

The Equilibrium Constant K and the Activities of Water Species.—Butler and his associates used the value K = 3.27 calculated by Topley and Eyring[11] for the equilibrium constant of the isotopic exchange reaction 2. No simple expression for the activities of water species can be obtained when $K \neq 4.00$. However, Orr and Butler[6] calculated the deviations in the water species activities resulting from K = 3.27 and expressed them in the form of a function $\phi(n)$. The activities of the water species, in terms of mole fraction, were calculated by the expressions

$$\left.\begin{array}{l} a_{H_2O} = (1 - n)^2\phi^2(1 - n) \\ a_{HDO} = 2n\phi(n) \times (1 - n)\phi(1 - n) \\ a_{D_2O} = n^2\phi^2(n) \end{array}\right\} \qquad (15)$$

Urey[12] has found the calculations of Topley and Eyring[11] to be incorrect and has obtained the value 3.96 for K (very nearly equal to the statistical value of 4.00). Thus, the equations 15 are incorrect. With K so very nearly equal to 4.00, the use of a function such as $\phi(n)$ in the expression of water species activities is unnecessary and the water activities, in terms of mole fractions, are given by

$$\left.\begin{array}{l} a_{H_2O} = (1 - n)^2 \\ a_{HDO} = 2n(1 - n) \\ a_{D_2O} = n^2 \end{array}\right\} \qquad (16)$$

The Equilibrium Constant, L.—The equilibrium constant L for reaction 4 cannot be directly measured. It can, however, be approximated by combining the free energy changes of electrolytic cell reactions and certain exchange reactions between hydrogen isotopes and isotopic water.

The free energy change of the reaction

$$H_2 + 2H_2O + 2D_3O^+ + 2Cl^-{}_{(D_2O)} \rightleftarrows D_2 + 2D_2O + 2H_3O^+ + 2Cl^-{}_{(H_2O)} \qquad (17)$$

can be obtained from the difference in the e.m.f.'s of the cells

$$\text{Pt, } H_2/HCl(m),\ H_2O/AgCl,\ Ag \qquad (18)$$

$$\text{Pt, } D_2/DCl(m),\ D_2O/AgCl,\ Ag \qquad (19)$$

at the same acid concentration.

By adding the free energy change for the reaction

$$D_2 + H_2O(l) \rightleftarrows H_2 + D_2O(l),\ K_{20} \qquad (20)$$

to that of reaction 17, the free energy change for the reaction

$$2D_3O^+ + 3H_2 + 2Cl^-{}_{(D_2O)} \rightleftarrows 2H_3O^+ + 3D_2O + 2Cl^-{}_{(H_2O)},\ L' \qquad (21)$$

can be obtained. Urey[12] has reported the equilibrium constants

$$D_2(g) + HDO(g) = D_2O(g) + HD(g),\ K_{22} \quad (22)$$

$$HD(g) + H_2O(g) = HDO(g) + H_2(g),\ K_{23} \quad (23)$$

over the temperature range 273.1–600°K. The equilibrium constant for reaction 20 can thus be calculated

$$K_{20} = K_{22} \times K_{23} \times \frac{P_{H_2O}}{P_{D_2O}} \quad (24)$$

At 25°, $K_{22} = 3.05$, $K_{23} = 3.70$ and P_{H_2O}/P_{D_2O} 1.16.[15] These data lead to $K_{20} = 12.9$. The equilibrium constant L' for reaction 17 can now be calculated from the expression

$$\log L' = \log K_{20} - \frac{2(E_D - E_H)}{0.05915} \quad (25)$$

where E_H and E_D represent the e.m.f.'s of cells (18) and (19), respectively.

It is of importance to notice that the free energy change of reaction 21 differs from the free energy change of reaction 4 by twice the free energy difference between (Cl^-) in H_2O and (Cl^-) in D_2O. The equilibrium constant L' therefore differs from the equilibrium constant L by the factor $(f_{Cl^-})^2{}_{H_2O}/(f_{Cl^-})^2{}_{D_2O}$, *i.e.*

$$L' = L \times \frac{(f_{Cl^-})^2_{H_2O}}{(f_{Cl^-})^2_{D_2O}} \quad (26)$$

Abel, Bratu and Redlich[16] reported a value of $(E_D - E_H) = -0.0034$ v. for cells (18) and (19) at 21° with $m_{HCl} = m_{DCl} = 0.1$. Noonan and LaMer[17] obtained $(E_D - E_H) = -0.0045$ v. for cells (18) and (19) at 25° with $m_{HCl} = m_{HCl} = 0.03 - 0.04$ and observed that the temperature coefficient of $(E_D - E_H)$ was very small over the temperature range 5–45°. The results of Abel, Bratu and Redlich lead to a value of $L' = 16.8$; those of Noonan and LaMer give $L' = 17.9$. Averaging the two results we obtain $L' = 17.4$.

Nelson and Butler[7] identified the equilibrium constant L' as L and thereby assumed that the activity coefficients of chloride ions were independent of the isotopic composition of the solvent. The value $L' = 15.3$ was used by these authors.

Schwarzenbach and his associates[13,14] have called attention to the effect of neglecting the free energy difference between the chloride ions in water and deuterium oxide. They made e.m.f. measurements with liquid junction cells of the type

$$Pt,\ H_2/HCl(C),\ H_2O//KCl\ (sat.),\ H_2O/Hg_2Cl_2,\ Hg$$

$$Pt,\ D_2/DCl(C),\ H_2O//KCl\ (sat.),\ H_2O/Hg_2Cl_2,\ Hg$$

at 20° with $C_{HCl} = C_{DCl} = 0.005\ M$ and obtained a potential of +0.0022 v. for the cell

$$Pt,\ H_2/HCl(C),\ H_2O//KCl\ (sat.),\ H_2O//\ D_2O,\ DCl(C)/D_2,\ Pt \quad (27)$$

The reaction for cell (27) is

$$\tfrac{1}{2}H_2 + H_2O(l) + D_3O^+ = H_3O^+ + D_2O(l) + \tfrac{1}{2}D_2 \quad (28)$$

The equilibrium constant for reaction 4 can now be calculated by the expression

$$\log L = \log K_{20} - \frac{2(0.0022)}{0.05915} \quad (29)$$

$$L = 11.0\ (\text{at } 25°)$$

This calculation of L depends on the assumption that the liquid junction potentials in cell (27) are equal in magnitude but opposite in sign. Since the transference numbers of most simple ions differ by not more than 2%,[18,19] it appears that the resultant liquid junction potential would be small. A rough approximation of the resulting liquid junction potential in cell (27) indicates that its neglect would cause an uncertainty of about 2% in L. In addition, this evaluation of L has assumed the temperature coefficient of cell (27) to be negligible. This appears to be a reasonable assumption, especially since the temperature coefficient of the quantity $(E_D - E_H)$ obtained from cells (18) and (19) was found to be negligibly small in this temperature range.

It appears, then, that the free energy difference between (Cl^-) in H_2O and (Cl^-) in D_2O is significant and that the activity coefficients cannot be considered independent of isotopic solvent composition. Having obtained this result, it is of importance to review the assumptions made concerning activity coefficients in the applications of the Gross–Butler theory.

(1) In all applications the activity coefficients, f_+, of the various lyonium ion species are assumed to be equal at a given acid concentration—and a given solvent composition.

(2) In the application to reaction rates it is assumed that the activity coefficient ratio $(f_+f_s)/f^*$ is independent of isotopic solvent composition (f_s represents the activity coefficient of substrate). When the reaction rates are those for substrate at unit activity, as was the case in equation 1, the assumption becomes: f_+/f_* is independent of n. (See ref. 9.)

(3) In the application of the theory to equilibria such as the thermodynamic distribution constants for picric acid between H_2O and D_2O–H_2O mixtures[3] and the thermodynamic dissociation constants for weak acids in D_2O–H_2O mixtures,[7,20,21] it is assumed that the activity coefficients of the ions and the undissociated acid are independent of isotopic solvent composition at a given acid concentration and a given ionic strength. Although this appears to be a dangerous assumption, it is possible that changes in the activity coefficients of undissociated acid species compensate for changes in the activity coefficients of ionic species as n is varied (*i.e.*, f_+f_-/f_s constant).

The $Q'(n)$ Function.—Butler and his associates[6,7] showed that experimental values of $Q'(n)$ calculated from equation 14 for acid-catalyzed reaction rates in D_2O–H_2O mixtures agreed generally well with $Q'(n)$-values derived from the distribution data for picric acid between H_2O and D_2O–H_2O mixtures and from the thermodynamic dissociation

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[15] R. L. Coombs, J. M. Googin and H. A. Smith, *J. Phys. Chem.*, **58**, 1000 (1954).

[16] E. Abel, E. Bratu and O. Redlich, *Z. physik. Chem.*, **173A**, 3600 (1935).

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[20] F. Brescia and V. K. LaMer, *ibid.*, **60**, 1963 (1938).

[21] C. K. Rule and V. K. LaMer, *ibid.*, **60**, 1975 (1938).

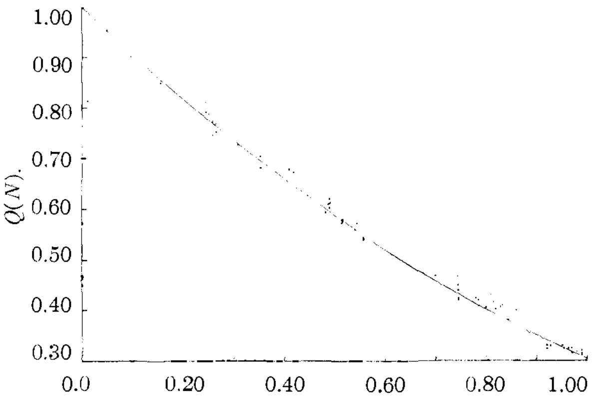

Fig. 1.

constants of weak acids in D_2O–H_2O mixtures. Furthermore, these authors obtained a theoretical expression for $Q'(n)$ by treating $k_2{}^{+H}$ and $k_3{}^{+H}$ of equation 11 as disposable parameters in obtaining a best-fit of the $Q'(n)$ function to the experimental data. Their method of fitting was based, however, on the value of $K = 3.27$ for the equilibrium constant of the exchange reaction 2 and on the value $L = 15.3$ for the equilibrium constant of reaction 4.

The available experimental data at 25° have been re-examined on the basis that $K = 4.0$ and $L = 11.0$ in order to establish what are considered to be more reliable values of the $Q'(n)$ function. Experimental values of $Q'(n)$ can be obtained from three types of measurements.

(1) Thermodynamic distribution constants for picric acid between H_2O and D_2O–H_2O mixtures[3]

$$Q'(n) = \left[(1 - n) + \frac{K_D}{(L)^{1/2}}\right] \Big/ K_n \qquad (30)$$

where K_n = the thermodynamic distribution constant of picric acid between H_2O and a D_2O–H_2O mixture of deuterium atom fraction n and K_D is the distribution constant of picric acid between H_2O and D_2O.

(2) Thermodynamic dissociation constants of weak acids as a function of deuterium content of the solvent[7,20,21]

$$Q'(n) = [(1 - n) + (K_H/K_D)n(L)^{1/2}](K_n/K_H) \qquad (31)$$

where the K's refer to the thermodynamic acid dissociation constants and the subscripts n, H and D refer to a D_2O–H_2O solvent mixture of deuterium atom fraction n, pure H_2O as solvent and pure D_2O as solvent, respectively.

(3) Thermodynamic rate constants of specifically acid-catalyzed reactions in which the proton transfer is non-rate-determining (*cf.* eq. 14 and eq. 16).

$$Q'(n) = [(1 - n) + (k_D/k_H)n(L)^{1/2}](k_H/k_n) \qquad (32)$$

where the symbols have the same meaning as those of equation 14.

Results and Discussion

Reaction Rates and Equilibria at 25°.—Fifty-eight values of $Q'(n)$ were calculated from experimental data at 25.0° representing two of the above types of measurements which included: the acid dissociation constants of acetic acid,[20] benzoic acid[21] and hydroquinone[21]; the acid-catalyzed reactions of ethyl orthoformate,[20] ethylene oxide,[10] epichlorohydrin[10] and β-methylglycidol[10]; and the acid-catalyzed bromination of acetone.[22]

In Fig. 1 the points represent the experimental $Q'(n)$-values and the curve represents the best fit of the $Q'(n)$ function obtained by adjusting $k_2{}^{+H} = 1.02$ and $k_3{}^{+H} = 1.37$ by the method of averages. At 25.0°, $Q'(n)$ is given by

$$Q'(n) = (1 - n)^3 + 2n(1 - n)^2\, 1.02 + n^2(1 - n)\, 1.37 + \frac{n^3}{(11.0)^{1/2}} \qquad (33)$$

These values obtained for $k_2{}^{+H}$ and $k_3{}^{+H}$ are to be compared with the values 1.05 and 1.10, respectively, which were determined in a similar manner by Orr and Butler.[6] The average deviation from the mean curve is 2.5% and the estimated standard deviation is 3.2%. Values of $Q'(n)$ obtained with eq. 33 are given in Table III.

Use of these new values for $Q'(n)$ leads to excellent agreement between observed and predicted reaction rates and acid dissociation constants. In Figs. 2 and 3 the solid curves represent reaction rates and acid dissociation constants calculated by the use of the new Gross–Butler functions obtained in this work and the broken curves represent the values calculated by the use of the former functions given by Nelson and Butler.[7] The improved fit in the case of hydroquinone is representative

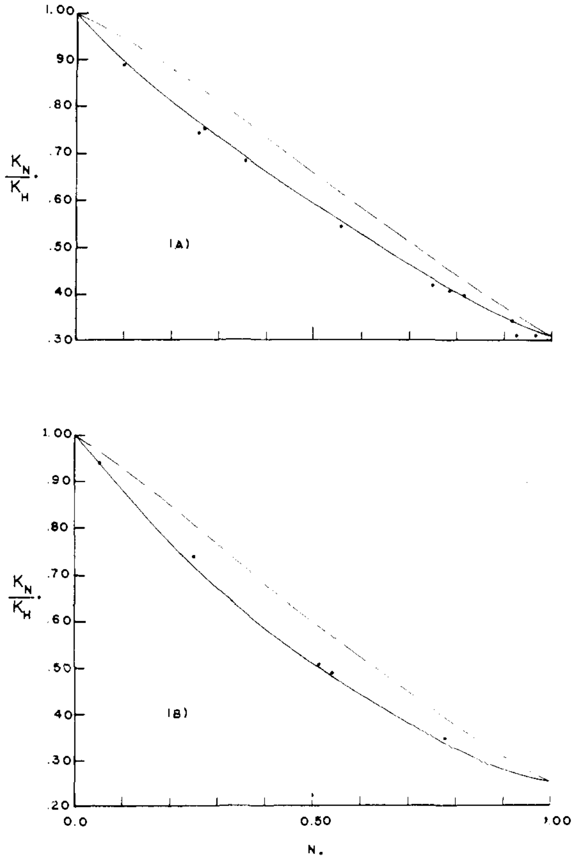

Fig. 2.—Acid dissociation constants in D_2O–H_2O mixtures at 25°: acetic acid[20] (a); and hydroquinone[21] (b).

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[22] O. Reitz, *Z. physik. Chem.*, **179A**, 119 (1937).

of that obtained for benzoic acid and the improvement in the case of ethylene oxide is representative of that obtained with epichlorohydrin and β-methylglycidol.

Fortunately, it is possible to check the parameters $k_2{}^{+\mathrm{H}}$ and $k_3{}^{+\mathrm{H}}$ of eq. 33 by an independent method. These equilibrium constants, as well as others of eq. 3, can be calculated from the equilibrium constant L alone. This equilibrium constant represents the fractionation factor for the distribution of hydrogen isotopes between lyonium ion and water.

$$L = \frac{(\mathrm{H_3O^+})^2(\mathrm{D_2O})^3}{(\mathrm{D_3O^+})^2(\mathrm{H_2O})^3} \qquad (34)$$

Since the equilibrium constant K for eq. 2 is virtually equal to the classical value of 4.0, the partition function for HDO is the geometric mean of the partition functions of H_2O and D_2O and the proportions of water species are determined by their symmetry numbers alone. This is an example of the rule of the geometric mean.[23] The equilibrium constants $k_2{}^{+\mathrm{H}}, k_3{}^{+\mathrm{H}}, k_1{}^{+\mathrm{D}}$ and $k_2{}^{+\mathrm{D}}$ can be calculated by applying the rule of the geometric mean to the distribution of hydrogen isotopes between the various lyonium ion and water species. This amounts to: (a) raising L to the appropriate power in order to obtain the desired partition function ratio, and (b) multiplying this result by the appropriate ratio of symmetry numbers.

As an example, let us calculate the equilibrium constant $k_2{}^{+\mathrm{H}}$

$$k_2{}^{+\mathrm{H}} = \frac{(\mathrm{H_2DO^+})}{(\mathrm{H^+})(\mathrm{HDO})} = \frac{(\mathrm{H_2DO^+})(\mathrm{H_2O})}{(\mathrm{H_3O^+})(\mathrm{HDO})} = \frac{3}{2} \times \frac{Q_{\mathrm{H_2DO^+}}Q_{\mathrm{H_2O}}}{Q_{\mathrm{H_3O^+}}Q_{\mathrm{HDO}}}$$

The symmetry numbers of H_2DO^+, H_3O^+, H_2O and HDO are 1, 3, 2 and 1, respectively. The Q's represent partition functions.

$$L^{-1/2} = \frac{Q_{\mathrm{D_3O^+}}Q_{\mathrm{H_2O}}{}^{3/2}}{Q_{\mathrm{H_3O^+}}Q_{\mathrm{D_2O}}{}^{3/2}}$$

By the rule of the geometric mean

$$\frac{Q_{\mathrm{H_2DO^+}}}{Q_{\mathrm{H_3O^+}}} = \left(\frac{Q_{\mathrm{D_3O^+}}}{Q_{\mathrm{H_3O^+}}}\right)^{1/3}$$

$$Q_{\mathrm{HDO}} = (Q_{\mathrm{H_2O}}\, Q_{\mathrm{D_2O}})^{1/2}$$

and we have

$$L^{-1/6} = \frac{Q_{\mathrm{H_2DO^+}}(Q_{\mathrm{H_2O}})^{1/2}}{Q_{\mathrm{H_3O^+}}(Q_{\mathrm{D_2O}})^{1/2}} = \frac{Q_{\mathrm{H_2DO^+}}Q_{\mathrm{H_2O}}}{Q_{\mathrm{H_3O^+}}Q_{\mathrm{HDO}}}$$

$$\therefore\ k_2{}^{+\mathrm{H}} = 3/2\, L^{-1/6} \qquad (35)$$

In a similar manner we obtain

$$k_3{}^{+\mathrm{H}} = 3L^{-1/3} \qquad (36)$$

$$k_1{}^{+\mathrm{D}} = 3L^{1/3} \qquad (37)$$

$$k_2{}^{+\mathrm{D}} = 3/2\, L^{1/6} \qquad (38)$$

At 25.0°, with $L = 11.0$

$$k_2{}^{+\mathrm{H}} = 1.01$$

$$k_3{}^{+\mathrm{H}} = 1.35$$

$$k_1{}^{+\mathrm{D}} = 6.67$$

$$k_2{}^{+\mathrm{D}} = 2.24$$

The above values calculated for $k_2{}^{+\mathrm{H}}$ and $k_3{}^{+\mathrm{H}}$ are in excellent agreement with those obtained empirically, the empirical values being $k_2{}^{+\mathrm{H}} = 1.02$ and $k_3{}^{+\mathrm{H}} = 1.37$.

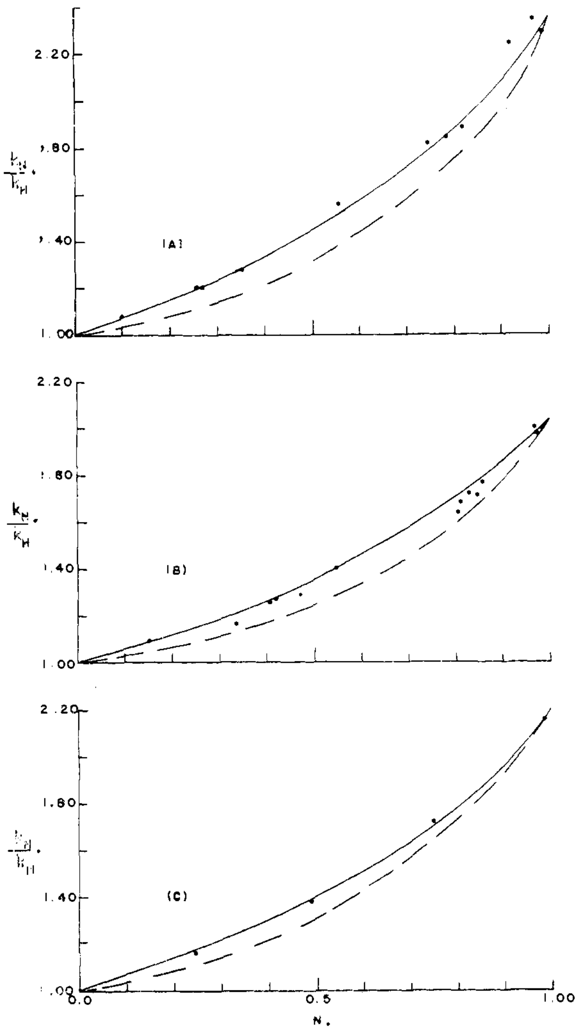

Fig. 3.—Acid-catalyzed reaction rates in D_2O-H_2O mixtures at 25°: hydrolysis of ethyl orthoformate[20] (a); bromination of acetone[22] (b); and hydrolysis of ethylene oxide[10] (c).

It is now important to examine the nature of the results that would be obtained if it were assumed that $L = L' = 17.4$. $Q'(n)$-values calculated (with $L = 17.4$) from the experimental data used to obtain Fig. 1 exhibit a scatter which is threefold that of the points in Fig. 1. Furthermore, these data lead to empirical values (determined as before) of 0.64 and 1.50 for $k_2{}^{+\mathrm{H}}$ and $k_3{}^{+\mathrm{H}}$ which *do not agree* at all well with the values $k_2{}^{+\mathrm{H}} = 0.93$ and $k_3{}^{+\mathrm{H}} = 1.16$, calculated by eq. 35 and 36.

The ability of the Gross–Butler theory to correctly predict reaction rates and acid dissociation constants plus the fact that the empirical values obtained for the equilibrium constants $k_2{}^{+\mathrm{H}}$ and $k_3{}^{+\mathrm{H}}$ agree with those calculated by eq. 35 and 36 when $L = 11.0$ lead us to believe that this value assigned to L is a reliable one.

In passing, it is noted that the relative acid dissociation constants for the lyonium ion species can be obtained by multiplying the equilibrium constants of eq. 3 by the appropriate *a priori* probability for obtaining proton or deuteron from the lyonium ion and taking the reciprocal of the result.

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[23] J. Bigeleisen, *J. Chem. Phys.*, **23**, 2264 (1955). See also ref. 12.

Furthermore, the relative basicities of the water species may be obtained provided that it is agreed to take the reciprocals of the acid dissociation constants of their respective conjugate acids as their basicity constants. Thus

$$\frac{K_{\mathrm{B.D_2O}}}{K_{\mathrm{B.H_2O}}} = \frac{1/3k_3{}^{+\mathrm{H}}}{k_1{}^{+\mathrm{H}}} = \frac{k_3{}^{+\mathrm{D}}}{1/3k_1{}^{+\mathrm{D}}} = L^{-1/3}\ (= 0.450 \text{ at } 25.0^\circ)$$

$$\frac{K_{\mathrm{B.HDO}}}{K_{\mathrm{B.H_2O}}} = \frac{2/3k_2{}^{+\mathrm{H}}}{k_1{}^{+\mathrm{H}}} = \frac{2/3k_3{}^{+\mathrm{D}}}{1/3k_1{}^{+\mathrm{D}}} = L^{-1/6}\ (= 0.671 \text{ at } 25.0^\circ)$$

Reaction Rates and Equilibria at Temperatures other than 25°.—If the temperature coefficient of the e.m.f. for cell (27) is negligibly small, then the temperature dependence of the equilibrium constant L arises almost wholly from that of the equilibrium constant K_{20} (*cf.* eq. 29). The data of Urey[12] for the equilibrium constants K_{22} and K_{23} plus the vapor pressure data for D_2O and H_2O permit the calculation of K_{20} at various temperatures. The results of such calculations are given in Table I. Values of the equilibrium constants L, $k_2{}^{+\mathrm{H}}$ and $k_3{}^{+\mathrm{H}}$ at various temperatures are given in Table II and values of the $Q'(n)$ function at different temperatures are listed in Table III.

TABLE I

EQUILIBRIUM CONSTANTS K_{22},[a] K_{23}[a] AND K_{20}[c] AT DIFFERENT TEMPERATURES

t, °C.	$K_{22} = \frac{(\mathrm{D_2O,g})(\mathrm{HD})}{(\mathrm{HDO,g})(\mathrm{D_2})}$	$K_{23} = \frac{(\mathrm{HDO,g})(\mathrm{H_2})}{(\mathrm{H_2O,g})(\mathrm{HD})}$	$\left(\frac{P_{\mathrm{H_2O}}}{P_{\mathrm{D_2O}}}\right)^b$	$K_{20} = \frac{(\mathrm{D_2O,l})(\mathrm{H_2})}{(\mathrm{H_2O,l})(\mathrm{D_2})}$
0.0	3.46	4.29	1.24	18.4
15.0	..	..	1.18	14.5
25.0	3.05	3.70	1.16	13.1
35.0	..	..	1.14	11.6
50.0	..	..	1.11	9.9
127.0	2.14	2.46	..	..

[a] Ref. 12. [b] Obtained by plotting log $(P_{\mathrm{H_2O}}/P_{\mathrm{D_2O}})$, from the data of ref. 15, *vs.* $1/T$. [c] Calculated by eq. 28; $(K_{22}\cdot K_{23})$-values at intermediate temperatures were obtained from a plot of log $(K_{22}\cdot K_{23})$ *vs.* $1/T$.

TABLE II

VALUES OF THE EQUILIBRIUM CONSTANTS L, $k_2{}^{+\mathrm{H}}$ AND $k_3{}^{+\mathrm{H}}$ AT DIFFERENT TEMPERATURES

t, °C.	L^a	$k_2{}^{+\mathrm{H}b}$	$k_3{}^{+\mathrm{H}c}$
0.0	15.4	0.95	1.21
15.0	12.3	0.99	1.30
25.0	11.0	1.01 (1.02)[d]	1.35 (1.37)[d]
35.0	9.8	1.03	1.40
50.0	8.4	1.05	1.47

[a] Calculated by eq. 29 using the values of K_{20} from Table I. [b] Calculated by eq. 35. [c] Calculated by eq. 36. [d] Empirical value determined in this work.

Reaction rates and equilibria measured in D_2O–H_2O mixtures at temperatures other than 25° include: the acid-catalyzed decomposition of diazoacetic ester at 0° and at 15°,[24] the acid dissociation constant of formic acid at 15°,[6] the acid-catalyzed hydrolysis of acetal at 15°,[6] the distribution constants for picric acid between H_2O and D_2O–H_2O mixtures at 18°,[3] the acid-catalyzed hydration of trimethylethylene at 30.0°[9] and 1-methylcyclopentene-1 at 35.20°[9]; and the acid-catalyzed hydrolysis of methyl acetate at 15°[7] and ethyl formate at 15°.[7] In Figs. 4 and 5 the solid curves represent rates and equilibria predicted by the Gross–Butler theory using $Q'(n)$- and L-values at the appropriate temperatures. The broken curves represent values calculated by using the functions of Nelson and Butler.[7] In general, application of the $Q'(n)$- and L-values corresponding to the temperatures at which the reaction rates and equilibrium constants were measured leads to very good agreement between the predicted and observed results. In the cases of the olefins, Fig. 5-c, -d, the rates are not predicted with the apparent high degree of precision that was obtained with the former values of $Q'(n)$ and L[9]; however, the agreement between the predicted and observed results is very satisfactory.

TABLE III

VALUES OF THE $Q'(n)$ FUNCTION AT DIFFERENT TEMPERATURES

	t, °C.				
n	0.0	15.0	25.0	35.0	50.0
0.00	1.00	1.00	1.00	1.00	1.00
.10	0.894	0.901	0.907	0.909	0.913
.20	.796	.809	.819	.823	.831
.30	.705	.724	.737	.743	.754
.40	.622	.644	.661	.668	.682
.50	.546	.571	.589	.598	.614
.60	.476	.503	.522	.532	.552
.70	.412	.441	.461	.472	.494
.80	.354	.384	.404	.417	.440
.90	.302	.333	.351	.366	.390
.95	.282	.309	.326	.346	.367
1.00	.255	.286	.303	.319	.345

Deviations from the Gross–Butler Rate Equation.—The rates of reactions which do not involve a fast equilibrium with lyonium ion prior to the rate-determining step, *i.e.*, those reactions in which proton transfer is rate determining, would not be expected to assume the unique character of the Gross–Butler rate equation. On the other hand, the rates of reactions following the pre-equilibrium mechanism may deviate from the rate equation because of the limitations of the assumptions in the Gross–Butler theory. It is quite difficult to predict these deviations; however, the two most troublesome assumptions appear to be: (a) that the activity coefficient ratio $(f_s f_+/f_*)$ is independent of solvent composition; and (b) that the specific rate constants for the reactions in pure H_2O and pure D_2O are independent of solvent composition (*cf.* ref. 9).

A rough estimate of the effects of variation in the activity coefficient ratio with solvent composition can be obtained by assuming that it varies linearly with solvent composition; *i.e.*

$$\left(\frac{f_s f_+}{f_*}\right)_n = \left(\frac{f_s f_+}{f_*}\right)_{\mathrm{H}} (1 + An)$$

$$A = \left[\left(\frac{f_s f_+}{f_*}\right)_{\mathrm{D}} \Big/ \left(\frac{f_s f_+}{f_*}\right)\right] - 1$$

where the subscripts refer to solvent composition. The Gross–Butler equation then takes the form

$$\frac{k_n}{k_{\mathrm{H}}} = \left[(1 - n) + \frac{k_{\mathrm{D}}/k_{\mathrm{H}}}{(R_f)_{\mathrm{D}}}\frac{n}{L^{1/3}}\right]\frac{(R_f)_n}{Q'(n)} \qquad (39)$$

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(24) Ph. Gross, H. Steiner and F. Krauss, *Trans. Faraday Soc.*, **34**, 351 (1938).

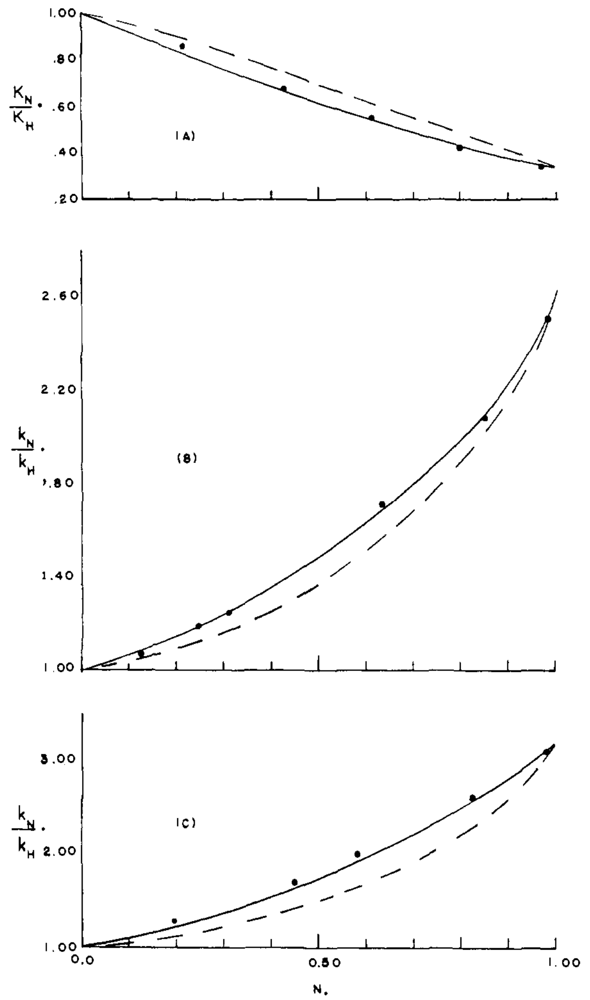

Fig. 4.—Application of the Gross–Butler theory to data obtained at 15°. Acid dissociation constants for formic acid[6] (a); acid-catalyzed hydrolysis of acetal[6] (b); and decomposition of diazoacetic ester[24] (c).

in which

$$(R_f)_n = \left(\frac{f_s f_+}{f_*}\right)_n \Big/ \left(\frac{f_s f_+}{f_*}\right)_H$$

$$(R_f)_D = \left(\frac{f_s f_+}{f_*}\right)_D \Big/ \left(\frac{f_s f_+}{f_*}\right)_H$$

Comparison of results calculated by eq. 39 with those calculated by eq. 14 at different (k_D/k_H)-ratios and various values of $(R_f)_D$ suggests that:

(1) Deviations from eq. 14 arising from variations in R_f vary with the (k_D/k_H)-ratio.

(2) Systematic positive deviations result when $(R_f)_D > 1$.

(3) Values of $(R_f)_D \leqslant 1$ result in: (a) systematic, positive deviations when $K_D/k_H > 2.00$; (b) systematic, negative deviations when $k_D/k_H \leqslant 1.0$; (c) either positive or negative deviations when $1 \leqslant k_D/k_H \leqslant 2$.

(4) The lower the (k_D/k_H)-ratio, the more sensitive are the rates to variation in R_f. The maximum difference between eq. 39 and 14 is *ca.* 5% when

$0.5 \leqslant (R_f)_D \leqslant 1.5,\ k_D/k_H = 3.0$
$0.5 \leqslant (R_f)_D \leqslant 1.3,\ k_D/k_H = 2.0$
$0.8 \leqslant (R_f)_D \leqslant 1.1,\ k_D/k_H = 1.0$
$0.9 \leqslant (R_f)_D \leqslant 1.1,\ k_D/k_H = 0.75$

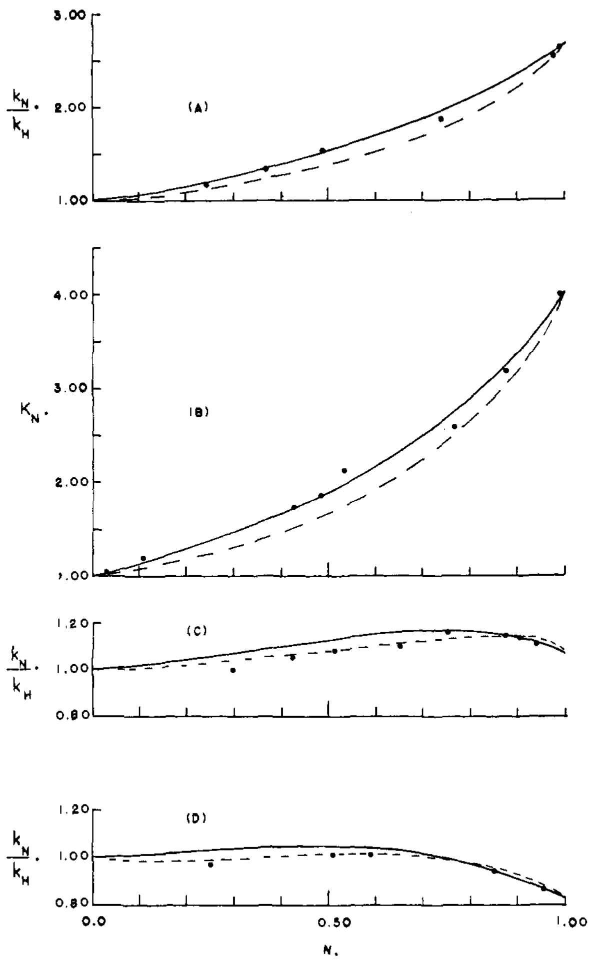

Fig. 5.—Application of the Gross–Butler theory to data obtained at various temperatures. The acid-catalyzed decomposition of diazoacetic ester at 0.0°[2] (a); distribution constants for picric acid between H_2O and D_2O–H_2O mixtures at 18°[8] (b); acid-catalyzed hydration of 1-methylcyclopentene-1 at 35.20°[9] (c); and trimethylethylene at 30.00°[9] (d).

In Fig. 6, the results obtained with eq. 39 and 14 are compared.

It is plausible that the variation in R_f arises mainly from changes in the activity coefficient of substrate, f_s, with solvent composition since f_+/f_* is the ratio of activity coefficients of two singly charged species and may vary little with solvent composition.

Assumption (b) requires that any exchangeable hydrogens on the substrate have no effect on the reaction rate. Furthermore, this assumption requires, in the case of an A-2 type mechanism involving the water molecule in the rate determining step, that the effects of the various solvent species on the reaction rate be virtually equivalent. Since the various water species differ in their relative basicities, it seems very probable that they would differ in their relative nucleophilicities.

In the case of the acid-catalyzed mutarotation of glucose[25] the observed rates are nearly linear and fall far below those predicted by the Gross–Butler rate equation as shown in Fig. 7-(a). Challis,

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[25] W. H. Hamill and V. K. LaMer, *J. Chem. Phys.*, **4**, 395 (1936).

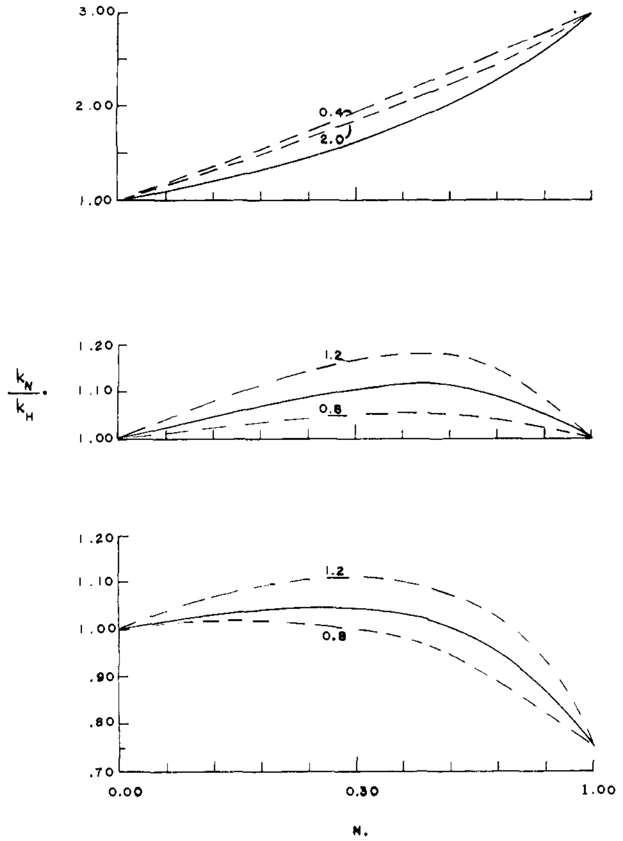

Fig. 6.—Estimates of deviations from the Gross–Butler rate equation arising from variation in activity coefficients with solvent composition. The broken curves show calculations with eq. 39 using the values shown near the curves for $(R_f)_D$; the solid curves were obtained with eq. 14.

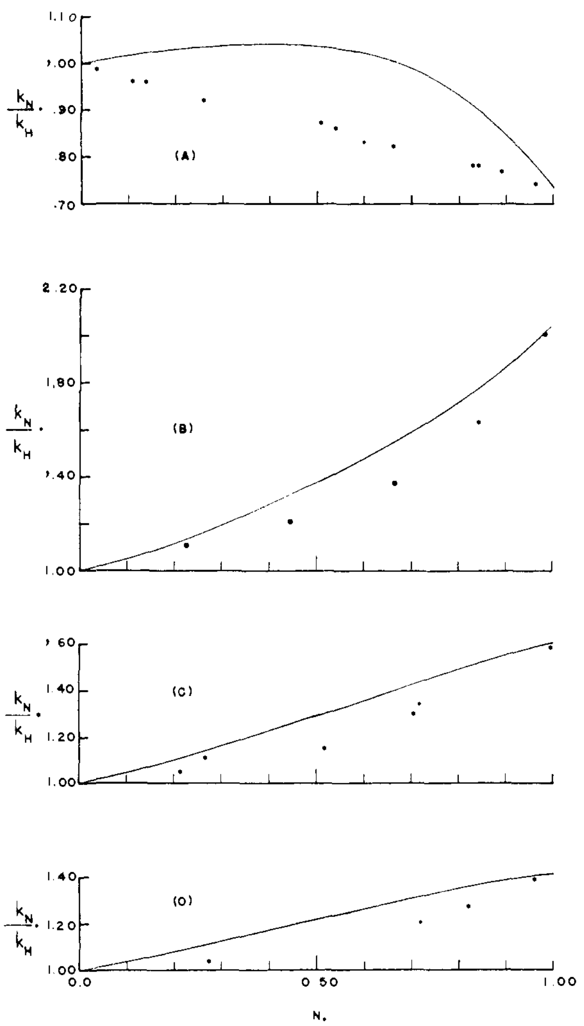

Fig. 7.—Acid-catalyzed reaction rates for: the mutarotation of glucose at 25°[25] (a); the inversion of sucrose at 25°[4] (b); the hydrolysis of methyl acetate at 15° (c), and ethyl formate at 15° (d).[7]

Long and Pocker[26] have suggested that acid-catalyzed mutarotation very probably proceeds *via* the pre-equilibrium mechanism and that the substitution of deuterium for hydrogen on the -OH group at the reaction center is largely responsible for the observed (k_D/k_H)-ratio which is less than unity. It is well known that the hydroxyl-hydrogens of glucose and tetramethylglucose exchange readily with deuterium of D_2O.[27,28] Hamill and Freudenburg found the equilibrium constant for H–D exchange between tetramethylglucose and HDO to be 0.83 (single measurement) and found that for glucose (with five exchangeable hydrogens) to be 0.69.[28] These data were calculated on the basis of $K = 3.27$ for reaction 2. Re-calculation, with $K = 4.0$, results in values of 0.78 and 0.63 for tetramethylglucose and glucose, respectively.

Hamill and LaMer[29] obtained an empirical value for the exchange equilibrium constant for glucose–HDO from the water-catalyzed mutarotation of glucose in D_2O-H_2O. Again, these calculations were based on $K = 3.27$. Re-calculation of the data, with $K = 4.0$, results in a value of 0.75 for the exchange equilibrium constant. As pointed out by Hamill and LaMer,[29] this value agrees well with that found for tetramethyl-glucose (0.78) which contains a single exchangeable hydrogen; but not with the value of 0.63 obtained for glucose. This result is consistent with the hypothesis that in the water-catalyzed mutarotation the principal effect on the rate is due to the replacement of D for H on the -OH group at the reaction center, the isotope effect of the four exchangeable hydrogens remote from the reaction center being small.

A similar effect would be expected in the acid-catalyzed mutarotation. This appears to be borne out by the results of Challis, Long and Pocker[26] who found the (k_D/k_H)-ratio for the acid-catalyzed mutarotation of tetramethylglucose to be 0.76, while that for glucose is 0.73.[25]

For a reaction involving a substrate with a single exchangeable hydrogen which exerts an isotope effect on the reaction rate, the Gross–Butler equation can be written in the form

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[26] B. C. Challis, F. A. Long and Y. Pocker, *J. Chem. Soc.*, 4679 (1957).

[27] E. A. Moelwyn-Hughes, R. Klar and K. F. Bonhoeffer, *Z. physik. Chem.*, **169A**, 113 (1934).

[28] W. H. Hamill and W. Freudenburg, This Journal, **57**, 1427 (1935).

[29] W. H. Hamill and V. K. LaMer, *J. Chem. Phys.* **4**, 145 (1936).

$$\frac{k_n}{k_H} = \left\{\left[(1-n) + \left(\frac{k_D}{k_H}\right)_H \frac{n}{L^{1/2}}\right] F_H + \left[(1-n) + \left(\frac{k_D}{k_H}\right)_D \frac{n}{L^{1/2}}\right](1-F_H)\right\} \frac{1}{Q'(n)} \quad (40)$$

$$\left(\frac{k_D}{k_H}\right)_H = \left(\frac{k_D}{k_H}\right) - \text{ratio for un-deuterated substrate}$$

$$\left(\frac{k_D}{k_H}\right)_D = \left(\frac{k_D}{k_H}\right) - \text{ratio for completely exchanged substrate}$$

$$F_H = \text{fraction of un-deuterated substrate}$$

(A more general equation taking into account all the exchangeable hydrogens is given in Appendix II.) Examination of this equation reveals that the right-hand member is always *greater than or equal to* the quantity

$$\left[(1-n) + \left(\frac{k_D}{k_H}\right)_D \frac{n}{L^{1/2}}\right] \frac{1}{Q'(n)}$$

since $(k_D/k_H)_H \geqslant (k_D/k_H)_D$. Equation 40 would therefore predict values *at least as great* as those predicted by eq. 14 and represented by the solid curve in Fig. 7-(a). These results lead us to the conclusion that the acid catalyzed mutarotation *does not proceed via* the pre-equilibrium mechanism.

It is now of interest to examine the data for the acid-catalyzed mutarotation of glucose on the hypotheses that the substitution of D for H on the –OH group at the reaction center is responsible for the decrease in reaction rate and that the reaction rate is otherwise independent of solvent composition. The reaction rate in D_2O–H_2O mixtures can be expressed

$$k_n = k_H F_H + k_D(1 - F_H) \quad (41)$$

The exchange equilibrium constant for the reaction

$$ROH + HDO = ROD + H_2O,\ k_{42} \quad (42)$$

can be calculated from the experimental k_n-values by the equation

$$K_{42} = \frac{(H_2O)(k_H - k_N)}{(HDO)(k_n - k_D)} \quad (43)$$

The ratio $(H_2O)/(HDO)$ is obtained by using $K = 4.0$ for reaction 2. In Table IV it is seen that values calculated for the equilibrium constant K_{42} remain fairly constant. When the mean value, $K_{42} = 0.52$, is used in conjunction with eq. 41 for the prediction of the rates, the predicted rates are noted to be in excellent agreement with those observed (*cf.* Table IV). It should be mentioned that eq. 41 and the data for glucose are relatively insensitive to the value of the equilibrium constant K_{42}. A value of $K_{42} \backsimeq 0.7$ would predict the rates reasonably well. No correction was made in these calculations for the isotopic dilution of the solvent resulting from exchange with the substrate. In this case the effects appear to be small.

These results, along with the complete failure of the Gross–Butler theory to predict the reaction rates, support the hypothesis that the acid catalyzed mutarotation proceeds via a single step, rate-determining protonation as previously suggested.[30]

Three reactions heretofore considered as following the Gross–Butler rate equation exhibit significant deviations. These include the acid-catalyzed inversion of sucrose[4] at 25° and the acid-catalyzed hydrolysis of methyl acetate and ethyl formate at 15°. In Fig. 7-(b), -(c) and -(d) it is seen that the experimental rates fall well below those predicted by the Gross–Butler rate equation. In the case of sucrose, it appears doubtful that the sag in the experimental points arises because of the failure in assumption (a) above. The observed rates cannot be predicted by eq. 39. Furthermore, the character of the results cannot be accounted for by eq. 40 or its more general form given in Appendix II; neither can they be accounted for by eq. 41, which would predict a linear dependence of the rates on solvent composition. A plausible explanation appears to lie in the effects of isotopic dilution resulting from exchange between substrate and solvent. Sucrose, with eight exchangeable hydrogens, could give rise to an isotopic dilution amounting to several per cent. This would tend to display the experimental points to the left in Fig. 7-(b).

TABLE IV

THE ACID-CATALYZED MUTAROTATION OF GLUCOSE AT 25°[25]

		k_n (min.$^{-1}$)	
n	K_{42}	Obsd.	Calcd. by eq. 41, with $K_{42} = 0.52$
0.00	..	0.311	...
.035	0.68	.307	0.307
.108	.62	.300	.301
.145	.54	.298	.298
.265	.59	.286	.288
.507	.44	.271	.268
.51	.53	.267	.267
.54	.47	.267	.265
.60	.54	.259	.260
.663	.48	.256	.255
.831	.51	.241	.241
.840	.48	.241	.240
.890	.36	.240	.236
.960	.56	.230	.230
1.00	..	(.227) extrap.	
	.52 (mean)		

In the cases of methyl acetate and ethyl formate a similar sag in the experimental points is noted. However, these substances do not contain readily exchangeable hydrogens. It appears that the deviations cannot be accounted for by eq. 39; in this (k_D/k_H)-range, eq. 39 will not yield values as low as are observed. It is of some interest to note that these reactions are the only reactions studied in D_2O–H_2O mixtures which are known to proceed *via* an A-2 mechanism involving the water molecule.[31] They may possibly represent a failure of assumption (b) above. If D_2O and HDO were less nucleophilic than H_2O, such a sag in the experimental curve might well be expected.

These reactions exhibiting significant deviations from the simple Gross–Butler theory serve to remind us that, although the solvent isotope effect of deuterium is a valuable tool for investigating reaction mechanisms, the inferences drawn from the application of the Gross–Butler theory should be evaluated with discretion. It should be noted that incomplete information may often be obtained from the study of reaction rates in pure D_2O and

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[30] K. F. Bonhoeffer, *Trans. Faraday Soc.*, **34**, 252 (1938).

[31] R. P. Bell, A. L. Dowding and J. A. Noble, *J. Chem. Soc.*, 3106 (1955).

pure H_2O alone—it is important to study the character of the reactions in D_2O–H_2O mixtures as well.

Acknowledgments.—The author wishes to thank Dr. Robert W. Taft of The Pennsylvania State University and Dr. J. Bigeleisen of the Brookhaven National Laboratories for their helpful comments and discussions. He is particularly indebted to the latter for calling to his attention the application of the rule of the geometric mean to the fractionation of hydrogen isotopes between lyonium ion and water.

Appendix I

The actual liquid junctions employed in cell (25) were very probably intermediate between the two extremes represented by the Planck and Henderson models for liquid junctions.[32–34]

In the former, liquid junction is obtained through a porous plug which prevents bulk mixing of the two solutions but permits ionic diffusion. In the latter, liquid junction is attained by a rapid mixing of the surface layers, the diffusion of the ions being negligible during the short time of mixing.

For two solutions in the same solvent the e.m.f. at the liquid junction

$$\text{solution (1), } c_1//\text{solution(2), } c_2$$

$$c_1 << c_2$$

is given by

$$\text{(Planck Model)} \quad E_L = \left(\frac{U_2 - V_2}{U_2 + V_2}\right) \frac{RT}{F} \ln \left(\frac{c_1}{c_2}\right)$$

$$\text{(Henderson Model)}\ E_L = \left(\frac{U_2 - V_2}{U_2 + V_2}\right) \frac{RT}{F} \ln \left(\frac{U_1 + V_1}{U_2 + V_2}\right)$$

where

c_1 (c_2) = total concn. (in eq./l.) of electrolyte in soln. 1(2)

$U_1(U_2)$ = $\Sigma c_i\lambda_i$ over all cations in soln. 1(2)

$V_1(V_2)$ = $\Sigma c_i\lambda_i$ over all anions in soln. 1(2)

c_i = concn. of ith species in eq./l.

λ_i = equivalent conductance of ith species

The absolute values of the resultant liquid junction potentials in cell (25) are thus given by

$$\text{(Planck Model)} \left| E_L \right| = \left| \left(\frac{\lambda_{K^+,\ H_2O} - \lambda_{Cl^-,\ H_2O}}{\lambda_{K^+,\ H_2O} + \lambda_{Cl^-,\ H_2O}}\right) \frac{RT}{F} \ln \left(\frac{c_{HCl,\ H_2O}}{c_{DCl,\ H_2O}}\right) \right|$$

$$\text{(Henderson Model)} \left| E_L \right| = \left| \left(\frac{\lambda_{K^+,\ H_2O} - \lambda_{Cl^-,\ H_2O}}{\lambda_{K^+,\ H_2O} + \lambda_{Cl^-,\ H_2O}}\right) \frac{RT}{E} \ln \left[\left(\frac{c_{DCl,\ H_2O}}{c_{HCl,\ H_2O}}\right)\left(\frac{\lambda_{D^+,\ D_2O} + \lambda_{Cl^-,\ D_2O}}{\lambda_{H^+,\ H_2O} + \lambda_{Cl^-,\ H_2O}}\right)\right] \right|$$

If the normal concentration of HCl in H_2O is equal to that of DCl in D_2O, the resultant liquid junction potential according to the Planck Model is zero. Using the values[19,35]: $\lambda_{K^+,H_2O} = 69.3$, $\lambda_{Cl^-H_2O} = 72.1$, $\lambda_{D^+,D_2O} = 239.5$, $\lambda_{H^+,H_2O} = 339.8$, $\lambda_{Cl^-,D_2O} = 62.6$ in the Henderson equation, the absolute value of the liquid junction potential in cell (25) is calculated to be 0.0002 v. at 25°. It would therefore be expected that this value represents the maximum liquid junction potential in cell (25). This uncertainty in E^0 for cell (25) would lead to an uncertainty of ± 0.2 in the value of 11.0 calculated for L by equation 29. Strictly speaking, the Planck and Henderson equations apply only when the solutions are in the same solvent; however, the bulk properties of H_2O and D_2O appear sufficiently similar to permit the use of the equations for the purpose of approximation. This treatment has assumed that any liquid junction potential due to $H_2O//D_2O$ is negligible.

Appendix II

For a reaction involving a substrate with *m*-exchangeable hydrogens which exert an isotope effect on the reaction rate, the Gross–Butler equation may be written

$$\frac{k_n}{k_H} = \sum_{i=0}^{m-1} \left[(1 - n) + \left(\frac{k_D}{k_H}\right)_i \frac{n}{L^{1/2}}\right] \frac{F_i}{Q'(n)} + \left[(1 - n) + \left(\frac{k_D}{k_H}\right)_D \frac{n}{L^{1/2}}\right] \frac{F_D}{Q'(n)}$$

where $(k_D/k_H)_i$ represents the (k_D/k_H)-ratio for the ith species in which i-H atoms have been replaced by D; (k_D/k_H) represents that for the completely exchanged species; F_i represents the fraction of the ith species: and F_D represents the fraction of the completely exchanged species.

The right hand member of the above equation is always greater than or equal to the quantity

$$\left[(1 - n) + \left(\frac{k_D}{k_H}\right)_D \frac{n}{L^{1/2}}\right] \frac{1}{Q'(n)}$$

since

$$\left(\frac{k_D}{k_H}\right)_i \geqslant \left(\frac{k_D}{k_H}\right)_D \text{ for all i}$$

Thus, the above equation will always predict values *at least as great* as those predicted by eq. 14.

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